

Markscheme

May 2018

Mathematics

Higher level

Paper 1

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a valid **Method**; working must be seen.
- (M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- (A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.
- R** Marks awarded for clear **Reasoning**.
- N** Marks awarded for **correct** answers if **no** working shown.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document “**Mathematics HL: Guidance for e-marking May 2018**”. It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp **A0** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, eg **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (eg substitution into a formula) and **A1** for using the **correct** values.
- Where the markscheme specifies (**M2**), **N3**, etc., do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final **A1**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct **FT** working shown, award **FT** marks as appropriate but do not award the final **A1** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final A1 (ignore the further working)
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final A1
3.	$\log a - \log b$	$\log(a - b)$	Do not award the final A1

3 **N** marks

Award **N** marks for **correct** answers where there is **no** working.

- Do **not** award a mixture of **N** and other marks.
- There may be fewer **N** marks available than the total of **M**, **A** and **R** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks (**d**)

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation **DM** should be used and a brief **note** written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives

$$f'(x) = (2\cos(5x - 3))5 \quad (=10\cos(5x - 3)) \quad \mathbf{A1}$$

Award **A1** for $(2\cos(5x - 3))5$, even if $10\cos(5x - 3)$ is not seen.

10 Accuracy of Answers

Candidates should **NO LONGER** be penalized for an accuracy error (**AP**).

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. $\cos \theta = \frac{(3i - 4j - 5k) \cdot (5i - 4j + 3k)}{|3i - 4j - 5k||5i - 4j + 3k|}$ **(M1)**

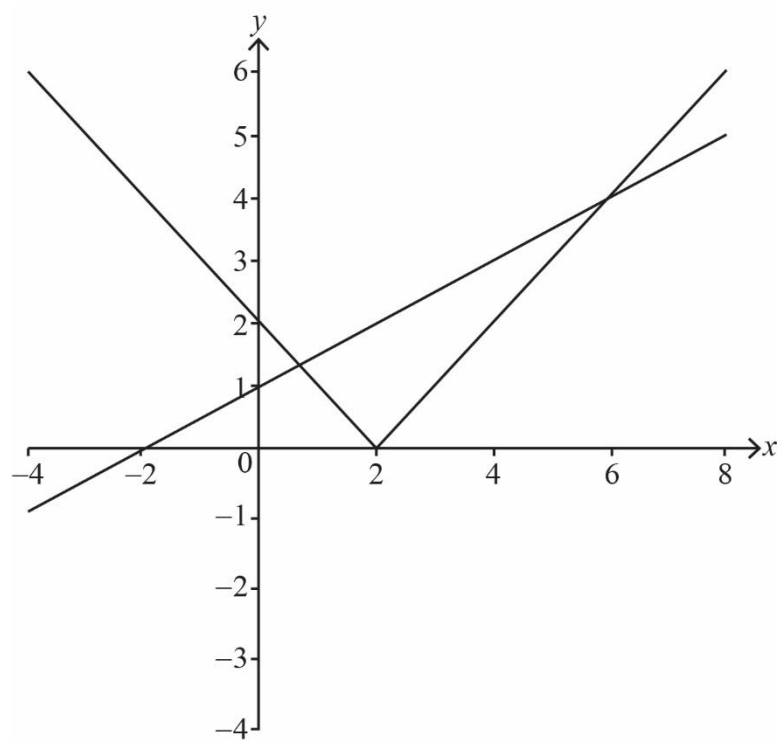
$= \frac{16}{\sqrt{50}\sqrt{50}}$ **A1A1**

Note: A1 for correct numerator and A1 for correct denominator.

$= \frac{8}{25} \left(= \frac{16}{50} = 0.32 \right)$ **A1**

[4 marks]

2. (a)



straight line graph with correct axis intercepts **A1**
 modulus graph: V shape in upper half plane **A1**
 modulus graph having correct vertex and y-intercept **A1**

[3 marks]

continued...

Question 2 continued

(b) **METHOD 1**

attempt to solve $\frac{x}{2} + 1 = x - 2$ **(M1)**

$x = 6$ **A1**

Note: Accept $x = 6$ using the graph.

attempt to solve (algebraically) $\frac{x}{2} + 1 = 2 - x$ **M1**

$x = \frac{2}{3}$ **A1**

[4 marks]

METHOD 2

$\left(\frac{x}{2} + 1\right)^2 = (x - 2)^2$ **M1**

$$\frac{x^2}{4} + x + 1 = x^2 - 4x + 4$$

$$0 = \frac{3x^2}{4} - 5x + 3$$

$$3x^2 - 20x + 12 = 0$$

attempt to factorise (or equivalent) **M1**

$$(3x - 2)(x - 6) = 0$$

$x = \frac{2}{3}$ **A1**

$x = 6$ **A1**

[4 marks]

Total [7 marks]

3. (a) equating sum of probabilities to 1 ($p + 0.5 - p + 0.25 + 0.125 + p^3 = 1$) **M1**

$$p^3 = 0.125 = \frac{1}{8}$$

$p = 0.5$ **A1**

[2 marks]

(b) (i) $\mu = 0 \times 0.5 + 1 \times 0 + 2 \times 0.25 + 3 \times 0.125 + 4 \times 0.125$ **M1**

$$= 1.375 \left(= \frac{11}{8} \right)$$
A1

continued...

Question 3 continued

(ii) $P(X > \mu) = P(X = 2) + P(X = 3) + P(X = 4)$ **(M1)**
 $= 0.5$ **A1**

Note: Do not award follow through **A** marks in (b)(i) from an incorrect value of p .

Note: Award **M** marks in both (b)(i) and (b)(ii) provided no negative probabilities, and provided a numerical value for μ has been found.

[4 marks]

Total [6 marks]

4. valid attempt to find $\frac{dy}{dx}$ **M1**

$$\frac{dy}{dx} = \frac{1}{(1-x)^2} - \frac{4}{(x-4)^2}$$
A1A1

attempt to solve $\frac{dy}{dx} = 0$ **M1**

$x = 2, x = -2$ **A1A1**

[6 marks]

5. (a) **METHOD 1**

state that $u_n = u_1 r^{n-1}$ (or equivalent) **A1**

attempt to consider a_n and use of at least one log rule **M1**

$$\log_2 |u_n| = \log_2 |u_1| + (n-1)\log_2 |r|$$
A1

(which is an AP) with $d = \log_2 |r|$ (and 1st term $\log_2 |u_1|$) **A1**

so A is an arithmetic sequence **AG**

Note: Condone absence of modulus signs.

Note: The final **A** mark may be awarded independently.

Note: Consideration of the first two or three terms only will score **MO**.

[4 marks]

continued...

Question 5 continued

METHOD 2

consideration of $(d =) a_{n+1} - a_n$ **M1**

$$(d) = \log_2 |u_{n+1}| - \log_2 |u_n|$$

$$(d) = \log_2 \left| \frac{u_{n+1}}{u_n} \right|$$
M1

$$(d) = \log_2 |r|$$
A1

which is constant **R1**

Note: Condone absence of modulus signs.

Note: the final **A** mark may be awarded independently.

Note: Consideration of the first two or three terms only will score **MO**.

(b) attempting to solve $\frac{3}{1-r} = 4$ **M1**

$$r = \frac{1}{4}$$
A1

$$d = -2$$
A1

[3 marks]

Total [7 marks]

6. (a) (i) attempt at product rule **M1**
 $f'(x) = -e^{-x} \sin x + e^{-x} \cos x$ **A1**

(ii) $g'(x) = -e^{-x} \cos x - e^{-x} \sin x$ **A1**

[3 marks]

(b) **METHOD 1**

Attempt to add $f'(x)$ and $g'(x)$ **(M1)**

$$f'(x) + g'(x) = -2e^{-x} \sin x$$
A1

$$\int_0^{\pi} e^{-x} \sin x \, dx = \left[-\frac{e^{-x}}{2} (\sin x + \cos x) \right]_0^{\pi} \text{ (or equivalent)}$$
A1

Note: Condone absence of limits.

$$= \frac{1}{2} (1 + e^{-\pi})$$
A1

continued...

Question 6 continued

METHOD 2

$$I = \int e^{-x} \sin x dx$$

$$= -e^{-x} \cos x - \int e^{-x} \cos x dx \text{ OR } = -e^{-x} \sin x + \int e^{-x} \cos x dx$$

M1A1

$$= -e^{-x} \sin x - e^{-x} \cos x - \int e^{-x} \sin x dx$$

$$I = -\frac{1}{2} e^{-x} (\sin x + \cos x)$$

A1

$$\int_0^\pi e^{-x} \sin x dx = \frac{1}{2} (1 + e^{-\pi})$$

A1

[4 marks]

Total [7 marks]

7. (a) $\frac{z + w}{z - w} = \frac{(a + c) + i(b + d)}{(a - c) + i(b - d)}$

$$= \frac{(a + c) + i(b + d)}{(a - c) + i(b - d)} \times \frac{(a - c) - i(b - d)}{(a - c) - i(b - d)}$$

M1A1

$$\text{real part} = \frac{(a + c)(a - c) + (b + d)(b - d)}{(a - c)^2 + (b - d)^2} \left(= \frac{a^2 - c^2 + b^2 - d^2}{(a - c)^2 + (b - d)^2} \right)$$

A1A1

Note: Award **A1** for numerator, **A1** for denominator.

[4 marks]

(b) $|z| = |w| \Rightarrow a^2 + b^2 = c^2 + d^2$

R1

hence real part = 0

A1

Note: Do not award **R0A1**.

[2 marks]

Total [6 marks]

8. (a) $\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$ (accept $du = \frac{1}{2} x^{-\frac{1}{2}} dx$ or equivalent)

A1

substitution, leading to an integrand in terms of u

M1

$$\int \frac{2u du}{u^3 + u} \text{ or equivalent}$$

A1

$$= 2 \arctan(\sqrt{x}) (+c)$$

A1

[4 marks]

continued...

Question 8 continued

$$(b) \frac{1}{2} \int_1^9 \frac{dx}{x^2 + \frac{1}{x^2}} = \arctan 3 - \arctan 1$$

A1

$$\tan(\arctan 3 - \arctan 1) = \frac{3 - 1}{1 + 3 \times 1}$$

(M1)

$$\tan(\arctan 3 - \arctan 1) = \frac{1}{2}$$

$$\arctan 3 - \arctan 1 = \arctan \frac{1}{2}$$

A1

[3 marks]

Total [7 marks]

Section B

9. (a) (i) a pair of opposite sides have equal length and are parallel
hence ABCD is a parallelogram **R1**
AG
- (ii) attempt to rewrite the given information in vector form **M1**
 $\mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{d}$ **A1**
 rearranging $\mathbf{d} - \mathbf{a} = \mathbf{c} - \mathbf{b}$ **M1**
 hence $\vec{AD} = \vec{BC}$ **AG**

Note: Candidates may correctly answer part i) by answering part ii) correctly and then deducing there are two pairs of parallel sides.

[4 marks]

(b) **EITHER**

use of $\vec{AB} = \vec{DC}$ **(M1)**

$$\begin{pmatrix} 2 \\ -3 \\ p+3 \end{pmatrix} = \begin{pmatrix} q+1 \\ 1-r \\ 4 \end{pmatrix}$$
A1A1

OR

use of $\vec{AD} = \vec{BC}$ **(M1)**

$$\begin{pmatrix} -2 \\ r-2 \\ 1 \end{pmatrix} = \begin{pmatrix} q-3 \\ 2 \\ 2-p \end{pmatrix}$$
A1A1

THEN

attempt to compare coefficients of $i, j,$ and k in their equation or statement to that effect **M1**
 clear demonstration that the given values satisfy their equation **A1**
 $p = 1, q = 1, r = 4$ **AG**

[5 marks]

- (c) attempt at computing $\vec{AB} \times \vec{AD}$ (or equivalent) **M1**
- $$\begin{pmatrix} -11 \\ -10 \\ -2 \end{pmatrix}$$
- A1**
- area = $\left| \vec{AB} \times \vec{AD} \right| (= \sqrt{225})$ **(M1)**
 = 15 **A1**

[4 marks]

continued...

Question 9 continued

(d) valid attempt to find $\vec{OM} \left(= \frac{1}{2}(\mathbf{a} + \mathbf{c}) \right)$ **(M1)**

$$\begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix} \quad \text{A1}$$

the equation is

$$\mathbf{r} = \begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix} + t \begin{pmatrix} 11 \\ 10 \\ 2 \end{pmatrix} \text{ or equivalent} \quad \text{M1A1}$$

Note: Award maximum **M1A0** if ' $\mathbf{r} = \dots$ ' (or equivalent) is not seen.

[4 marks]

(e) attempt to obtain the equation of the plane in the form $ax + by + cz = d$ **M1**
 $11x + 10y + 2z = 25$ **A1A1**

Note: **A1** for right hand side, **A1** for left hand side.

[3 marks]

(f) (i) putting two coordinates equal to zero **(M1)**
 $X\left(\frac{25}{11}, 0, 0\right), Y\left(0, \frac{5}{2}, 0\right), Z\left(0, 0, \frac{25}{2}\right)$ **A1**

(ii) $YZ = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{25}{2}\right)^2}$ **M1**

$$= \sqrt{\frac{325}{2}} \left(= \frac{5\sqrt{104}}{4} = \frac{5\sqrt{26}}{2} \right) \quad \text{A1}$$

[4 marks]

Total [24 marks]

10. (a) attempt to make x the subject of $y = \frac{ax+b}{cx+d}$ **M1**

$$y(cx+d) = ax+b \quad \text{A1}$$

$$x = \frac{dy-b}{a-cy} \quad \text{A1}$$

$$f^{-1}(x) = \frac{dx-b}{a-cx}, \quad \text{A1}$$

Note: Do not allow $y =$ in place of $f^{-1}(x)$.

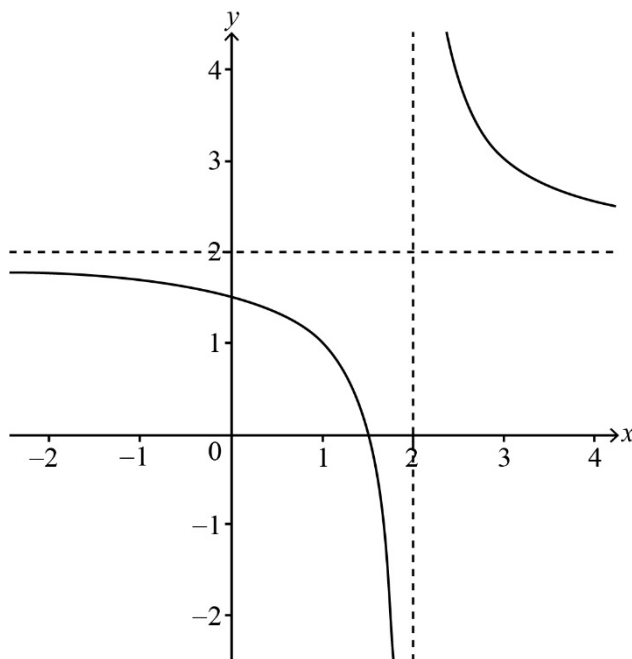
$$x \neq \frac{a}{c}, \quad (x \in \mathbb{R}) \quad \text{A1}$$

Note: The final **A** mark is independent.

[5 marks]

(b) (i) $g(x) = 2 + \frac{1}{x-2}$ **A1A1**

(ii)



hyperbola shape, with single curves in second and fourth quadrants and **A1**

third quadrant blank, including vertical asymptote $x = 2$ **A1**
 horizontal asymptote $y = 2$

intercepts $\left(\frac{3}{2}, 0\right), \left(0, \frac{3}{2}\right)$ **A1**

[5 marks]

continued...

Question 10 continued

(c) the domain of $h \circ g$ is $x \leq \frac{3}{2}, x > 2$

A1A1

the range of $h \circ g$ is $y \geq 0, y \neq \sqrt{2}$

A1A1

[4 marks]

Total [14 marks]

11. (a) **METHOD 1**

$$\begin{aligned} \log_{r^2} x &= \frac{\log_r x}{\log_r r^2} \left(= \frac{\log_r x}{2 \log_r r} \right) \\ &= \frac{\log_r x}{2} \end{aligned}$$

M1A1

AG

[2 marks]

METHOD 2

$$\begin{aligned} \log_{r^2} x &= \frac{1}{\log_x r^2} \\ &= \frac{1}{2 \log_x r} \\ &= \frac{\log_r x}{2} \end{aligned}$$

M1

A1

AG

[2 marks]

(b) **METHOD 1**

$$\log_2 y + \log_4 x + \log_4 2x = 0$$

$$\log_2 y + \log_4 2x^2 = 0$$

M1

$$\log_2 y + \frac{1}{2} \log_2 2x^2 = 0$$

M1

$$\log_2 y = -\frac{1}{2} \log_2 2x^2$$

$$\log_2 y = \log_2 \left(\frac{1}{\sqrt{2x}} \right)$$

M1A1

$$y = \frac{1}{\sqrt{2}} x^{-1}$$

A1

Note: For the final **A** mark, y must be expressed in the form px^q .

[5 marks]

continued...

Question 11 continued

METHOD 2

$$\log_2 y + \log_4 x + \log_4 2x = 0$$

$$\log_2 y + \frac{1}{2} \log_2 x + \frac{1}{2} \log_2 2x = 0$$

M1

$$\log_2 y + \log_2 x^{\frac{1}{2}} + \log_2 (2x)^{\frac{1}{2}} = 0$$

M1

$$\log_2 (\sqrt{2}xy) = 0$$

M1

$$\sqrt{2}xy = 1$$

A1

$$y = \frac{1}{\sqrt{2}}x^{-1}$$

A1

Note: For the final **A** mark, y must be expressed in the form px^q .

[5 marks]

(c) the area of R is $\int_1^{\alpha} \frac{1}{\sqrt{2}} x^{-1} dx$

M1

$$= \left[\frac{1}{\sqrt{2}} \ln x \right]_1^{\alpha}$$

A1

$$= \frac{1}{\sqrt{2}} \ln \alpha$$

A1

$$\frac{1}{\sqrt{2}} \ln \alpha = \sqrt{2}$$

M1

$$\alpha = e^2$$

A1

Note: Only follow through from part (b) if y is in the form $y = px^q$.

[5 marks]

Total [12 marks]